Calculate Balloon Ascent Rate

1. Force Balance and Buoyancy

The vertical movement of a helium-filled balloon depends on the sum of the various forces acting upon it, which we will fall the net force (F_{net}). These forces include the upward buoyancy force (F_B) of the inflated balloon, the downward gravitational force exerted upon the helium inside the balloon (F_{He}), and the downward gravitational force exerted upon the balloon itself (F_{ball}). This is written as

$$F_{net} = F_B - F_{He} - F_{ball} \qquad (1)$$

Within the atmosphere, the buoyancy force exerted on an object or substance (such as helium) is by definition equal to the weight of the air that the object or substance displaces. Weight is simply the gravitation force exerted upon an object or substance. Thus, the buoyancy force of the helium in the balloon is equal to the gravitational force exerted upon the amount of surrounding air that would otherwise occupy the space filled by the helium, if the balloon and helium were not there.

The gravitational force is given by

$$F = mg$$

where m is mass and g is the acceleration due to gravity. We also know that $m = \rho V$, where ρ is the density of the object or substance and V is its volume. By substitution, this means that

$$F = \rho V g$$

The upward buoyancy force of the balloon can therefore be written as

$$F_B = \rho_{air} V g$$

where ρ_{air} is the density of the surrounding air and V is the volume of air displaced by the balloon. By substituting this into equation (1), we get

$$F_{net} = (\rho_{air} V g) - F_{He} - F_{ball}$$

which is also equal to

$$F_{net} = (\rho_{air} V g) - (\rho_{He} V g) - (m_{ball} g)$$

where ρ_{He} is the density of the helium inside the balloon and m_{ball} is the mass of the balloon. Note that the volume of the helium and the volume displaced by the balloon are the same (if we neglect the very thin width of the balloon's skin). After some factoring, we arrive at

$$F_{net} = g[(\rho_{air} - \rho_{He})V - m_{ball}]$$

The above equation assumes that the balloon is not also lifting a payload. If a payload is attached to the balloon, the force of gravity acting upon it (F_{pay}) must also be included. Thus,

$$F_{net} = F_B - F_{He} - F_{ball} - F_{pay}$$

Going through the same steps as above, we finally arrive at

$$F_{net} = g\left[(\rho_{air} - \rho_{He})V - m_{ball} - m_{pay}\right] \qquad (2)$$

where m_{pay} is the mass of the payload. Thus, the greater the volume of helium in the balloon, the greater the buoyancy force to counteract the gravitational force pulling down on the balloon, the helium, and the payload.

2. Drag Force

As the balloon moves through the atmosphere, air resistance induces a drag force (F_D) that directly (or proportionately) opposes the net force—given by equation (2)—acting upon the balloon. The drag force equation for a balloon moving vertically through the air is written as

$$F_D = \frac{c_d \rho_{air} v^2 \pi {r_{ball}}^2}{2}$$

where c_d is the drag coefficient, v is the vertical velocity (i.e., the ascent rate) of the balloon, and r_{ball} is the radius of the balloon. Remember, the balloon must be moving for there to be a drag force! Since the drag force is balanced by the net force acting upon the balloon, we set $F_{net} = F_D$, which gives us

$$g\left[(\rho_{air} - \rho_{He})V - m_{ball} - m_{pay}\right] = \frac{c_d \rho_{air} v^2 \pi {r_{ball}}^2}{2} \qquad (3)$$

Before moving on, we can break down the equation even further by converting radius to diameter and substituting $V = (4/3)\pi r^3$, which gives us

$$g\left[(\rho_{air} - \rho_{He})\left(\frac{4\pi\left(\frac{D_{ball}}{2}\right)^3}{3}\right) - m_{ball} - m_{pay}\right] = \frac{c_d \rho_{air} v^2 \pi \left(\frac{D_{ball}}{2}\right)^2}{2}$$

where D_{ball} is the diameter of the balloon. This simplifies to

$$g\left[(\rho_{air} - \rho_{He})\left(\frac{\pi {D_{ball}}^3}{6}\right) - m_{ball} - m_{pay}\right] = \frac{c_d \rho_{air} v^2 \pi {D_{ball}}^2}{8}$$

We are now ready to solve for the vertical velocity *v*. Multiplying both sides by 8, rearranging the variables to isolate v^2 on one side of the equation, and taking the square root leaves us with the following equation for the ascent rate

$$v = \sqrt{\frac{8g\left[(\rho_{air} - \rho_{He})\left(\frac{\pi D_{ball}{}^3}{6}\right) - m_{ball} - m_{pay}\right]}{c_d \rho_{air} \pi D_{ball}{}^2}} \tag{4}$$

This tells us that if we know the mass of the balloon, mass of the payload, and diameter of the helium-inflated balloon at any given time, we can calculate the instantaneous ascent rate through the atmosphere. However, this assumes that the balloon is a perfect sphere, which it is not. Therefore, the calculation should be viewed as a good approximation.

3. Affect of Temperature and Density

It is also worth noting that the density of air and helium are not constant and in fact vary slightly with both pressure and temperature. This relationship for can be calculated, to a good approximation, with the Ideal Gas Law. This law can be written as

$$\rho = \frac{p}{\left(\frac{R}{M}\right)T}$$

where ρ is the density (kg m^{-3}), p is the pressure (Pa), R is the universal gas constant (8.314 J mole^{-1} K^{-1}), M is the molar mass (kg mole^{-1}), and T is the temperature (K). The molar mass is 0.02897 kg mole^{-1} for dry (unsaturated) air and 0.004 kg mole^{-1} for helium. Thus,

$$\rho_{air} = \frac{p_{air}}{287 \cdot T_{air}}$$

$$\rho_{He} = \frac{p_{He}}{2078.5 \cdot T_{He}}$$

When helium enters the balloon, we assume that its pressure immediately equalizes with the surrounding atmospheric pressure. We also assume that it eventually warms or cools to match the ambient temperature. Both changes will therefore cause a change in its density as well.

For example, if the ambient air pressure and temperature in Laramie are 77,000 Pa (770 mb) and 293.15 K (20°C), respectively, the air density will be 0.915 kg m^{-3}. Assuming the helium indeed reaches equilibrium with the surrounding environment, its density will be 0.126 kg m^{-3} once it enters the balloon. If you are instead at sea level, in Miami, where the ambient air pressure and temperature are perhaps 100,000 Pa (1000 mb) and 303.15 K (30°C), respectively, the air density will be 1.149 kg m^{-3} and the helium density will be 0.159 kg m^{-3}.

Do these density differences actually matter? To find out, let's calculate how much helium we get from one helium cylinder. A T-size (300 ft^3) helium cylinder, which has an internal volume of 1.76 ft^3, is initially filled with about 1.4 kg of helium gas to a pressure of about 2500 PSI. (This pressure varies slightly as a function of helium temperature, which is assumed to match the ambient temperature.) If you released all the helium from this cylinder into a balloon, it would expand to about 392 ft^3 in Laramie (at 77,000 Pa and 293.15 K) but only to about 313 ft^3 in Miami (at 100,000 Pa and 303.15 K). This is of course because the helium expands inside the balloon to equilibrate with the ambient atmospheric pressure, which is lower in Laramie and higher in Miami.

Now that we know how much helium one cylinder holds, let's find out how much of it gets used in various scenarios. Let's suppose you want your 1200-gram balloon to lift a 4500-gram payload with an ascent rate of 5 m s^{-1} immediately after launch. Using equation (4), in Laramie (at 77,000 Pa and 293.15 K) you would need to fill your balloon with enough helium to have a mean diameter at launch of 2.6 m, which equates to a helium volume of about 9.2 m^3 (325 ft^3). With a helium density in Laramie of 0.126 kg m^{-3}, this amounts to about 1.16 kg of helium, or ~82% of what was in the cylinder. To reach a launch ascent rate of 5 m s^{-1} in Miami (at 100,000 Pa and 303.15 K), equation (4) indicates that you would need a mean balloon diameter at launch of only 2.43 m, equating to a helium volume of about 7.5 m^3 (265 ft^3). With a helium density in Miami of 0.159 kg m^{-3}, this amounts to about 1.19 kg of helium, or ~84% of what was in the cylinder.

It therefore appears that a slightly larger amount of helium is necessary to inflate the balloon in Miami, all else being equal. The difference in this case is very small (2–3%), but not totally negligible.

4. Calculating Helium Lift

The previous exercise sought to find an equation to calculate the instantaneous ascent rate, if we already know the mass of the balloon, mass of the payload, and radius/diameter of the balloon (which is proportional to the amount of helium in the balloon). However, we usually know what we want our ascent rate to be. What we *actually* want to know is how much helium to put in the balloon. There needs to be enough helium to provide the proper amount of buoyancy to lift not only the balloon itself, but also the payload hanging below. Furthermore, it needs to lift the balloon and payload not just off the ground, but upwards at some desired ascent rate. This is the free lift.

This is a more challenging problem. To calculate the free lift force, we must solve equation (2). Although that seems easy enough, in this scenario we don't know the volume of the balloon since that is determined by how much helium is added. Thus, we must start by iteratively solving equation (4) using different values of D_{ball} until the solution for v is equal to the desired ascent rate. Once we have D_{ball}, we can calculate the volume of the balloon and solve equation (2).

For example, if we'd like to launch a 1200-gram balloon and 4500-gram payload from Laramie with a desired ascent rate of 5 m s^{-1}, assuming an air density of 0.915 kg m^{-3}, a helium density of 0.126 kg m^{-3}, and a drag coefficient of 0.25, we solve for equation (4). We find that when $v = 5$ m s^{-1}, $D_{ball} = 2.6$ m. Therefore, $r_{ball} = 1.3$ m and $V = 9.2$ m^3. We can then take V and plug it into equation (2) to calculate the free lift force. In this case, the free lift force ends up being 15.3 N. Dividing this by g gives us the free lift in terms of mass, which is 1.56 kg.

There are four measurements of lift: neutral lift, free lift, gross lift, and neck lift. *Neutral lift* simply refers to the amount of lift needed to raise the balloon and its payload off the ground, providing what is essentially neutral buoyancy. It is equal to the mass (or weight) of the entire balloon system, including the balloon itself and all payload items it is carrying below it. If let go (and in the absence of wind), the balloon and payload will float in place, neither rising nor sinking. *Free lift* refers to the extra lift needed for the balloon to be able to carry the payload upward at some ascent rate. *Gross lift* is the sum of the neutral lift and free lift, providing a measure of the total lift acting upon the balloon system by the helium.

When inflating a balloon, we use a measure called *neck lift*. With the neck of the balloon tethered to the ground during inflation to keep the balloon from flying away, we can attach a scale to the tether line to measure how much the balloon is "pulling upward", but only after it acquires enough lift to raise itself off the ground. The extra lift that is then measured by the scale is appropriately referred to as the neck lift (sometimes called nozzle lift), since it refers only to the amount of lift that will be acting upon the payload that hangs below the neck of the balloon. Neck lift is calculated by adding the free lift and the mass of the payload. When the scale measurement is equal to the neck lift, the balloon should hypothetically be inflated with just the right amount of helium to carry the payload upward at the target ascent rate. In the case described above, the neutral lift is 5.7 kg (balloon mass + payload mass), the free lift is 1.56 kg, the gross lift is 7.26 kg (neutral lift + free lift), and the neck lift is 6.06 kg (free lift + payload mass).